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Quark spin content of vector mesons

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η' dominance of form factors of the topological QCD current leads to an expression for the portion of the spin of the light vector mesons that is due to quarks. Vector dominance of the radiative decays of the η' is used to estimate the couplings of this meson to the vector ones. This results in the conclusion that only around 30% of the spin of these particles is due to quarks. Possible interpretations of this result are presented.

The European Muon Collaboration [1] results on polarized proton structure functions have sparked an interest in the fraction of the proton's spin born by quarks. A naive interpretation yielded the improbable result that none of the proton's spin was carried by the quarks and all the spin had to be in the gluon sector. This is modified when one takes the QCD chiral anomaly into account [2]. Veneziano [3] and Shore and Veneziano [3] obtained a Goldberger-Treiman-type relation for the proton's quark spin content. Efremov, Soffer, and Törnqvist [4,5] used such relations, together with an assumption on the large momentum behavior of a certain matrix element and reasonable estimate for the η'-nucleon coupling constant, to show that most of the proton's spin is carried by quarks. In this work we obtain a similar relation for the quark spin content of vector mesons. For the ρ and ω mesons, vector dominance applied to radiative η' decays provides us with the relevant coupling constants, and a bound on the rate for $\phi \rightarrow \eta'\gamma$ gives a limit on the η'-ϕ coupling. *With the above assumptions, we find that the quarks carry only about 30% of the spin of these particles.*

First we briefly review the results of Refs. [3,4,5]. The usual flavor-singlet, axial-vector current is

$$j_\mu^5 = \sum_{f=1}^{N_f} \bar{q}_f \gamma_\mu \gamma_5 q_f \ , \tag{1}$$

while k_μ is the gauge-dependent topological current whose divergence is Q:

$$Q = \frac{\alpha_s}{8\pi} N_f G_{\alpha,\mu\nu} \tilde{G}_\alpha^{\mu\nu} \ ; \tag{2}$$

N_f is the number of flavors and G is the gauge field strength tensor. In the chiral limit we have

$$\partial^\mu j_\mu = Q \ . \tag{3}$$

The matrix elements of these currents between proton states are

$$\begin{aligned} \langle p',s'|j_\mu^5|p,s\rangle &= \bar{u}(p',s')[\gamma_\mu\gamma_5 G_1^{(P)}(q^2) + q_\mu\gamma_5 G_2^{(P)}(q^2)]u(p,s) \ , \\ \langle p',s'|k_\mu^5|p,s\rangle &= \bar{u}(p',s')[\gamma_\mu\gamma_5 \tilde{G}_1^{(P)}(q^2) + q_\mu\gamma_5 \tilde{G}_2^{(P)}(q^2)]u(p,s) \ . \end{aligned} \tag{4}$$

This form for the matrix element for k_μ^5 is valid in any covariant gauge. A consequence of the gauge variance of the topological current is that the form factor $\tilde{G}_2^{(P)}(q^2)$ has a ghost pole at $q^2=0$ [6]. It is the gauge-invariant residue of this pole that determines the quark content of the proton's spin $\Delta\Sigma^P$ [2]:

$$\Delta\Sigma^P = G_1^{(P)}(0) - \tilde{G}_1^{(P)}(0) = \frac{q^2\tilde{G}_2^{(P)}(q^2)|_{q^2=0}}{2M_P} \ ; \tag{5}$$

M_P is the proton's mass and the second equality is due to Eq. (3). $q^2\tilde{G}_2^{(P)}(q^2)$ has a pole at $m_{\eta'}^2$; it may also have a constant contribution from the direct coupling of the ghost pole to nucleons [3,5]. The presence or absence of this direct ghost coupling is equivalent to the question of whether $q^2\tilde{G}_2^{(P)}(q^2)$ satisfies a subtracted or unsubtracted dispersion relation. The last statement requires some care as even though $q^2\tilde{G}_2^{(P)}(q^2)|_{q^2=0}$ is gauge invariant it is not invariant away from $q^2=0$. Changing gauges, at least within the class of covariant gauges, adds polynomials in q^2, which vanish at $q^2=0$, to $q^2\tilde{G}_2^{(P)}(q^2)$; we may consider a dispersion relation for $q^2\tilde{G}_2^{(P)}(q^2)$ in the gauge in which it has at most a constant behavior for large q^2. The usual Goldberger-Treiman relation is based on the assumption that the divergence of the isospin axial-vector current is soft at high momenta and satisfies an unsubtracted dispersion relation; it is thus dominated by the pion pole. In Ref. [3] a similar assumption was made for $\tilde{G}_2^{(P)}(q^2)$, i.e., that it is unsubtracted and dominated by the η' pole. The quark content of the proton's spin was found to be

$$\Delta\Sigma^P = \frac{\sqrt{2N_f} f_{\eta'}}{2M_P} g_{\eta' NN} \ , \tag{6}$$

where $f_{\eta'}$ is the η' decay constant (normalization is such that $f_\pi=93$ MeV) and $g_{\eta' NN}$ is the η'-nucleon coupling constant. Estimates for $f_{\eta'}$ [7] and for $g_{\eta' NN}$ yield $\Delta\Sigma^P \approx 1$. Corrections due to isospin and SU(3) breaking in the quark masses [5] were found to be small.

We shall now extend these ideas to the light vector mesons ρ, ω, and ϕ. The matrix elements between any of these meson states with momenta p,p' and helicities λ,λ' of the singlet axial-vector and topological currents are defined as

$$\begin{aligned}\langle p',\lambda'|j_\mu^5|p,\lambda\rangle &= -i\epsilon^{\alpha\nu\rho\sigma}\epsilon_V^*(\lambda')\epsilon_\sigma(\lambda)(p+p')_\nu \\ &\times[g_{\alpha\mu}G_1^{(V)}(q^2) \\ &\quad +(p-p')_\alpha(p-p')_\mu G_2^{(V)}(q^2)]+\cdots ,\end{aligned} \tag{7}$$

$$\begin{aligned}\langle p',\lambda'|k_\mu|p,\lambda\rangle &= -i\epsilon^{\alpha\nu\rho\sigma}\epsilon_V^*(\lambda')\epsilon_\sigma(\lambda)(p+p')_\nu \\ &\times[g_{\alpha\mu}\tilde{G}_1^{(V)}(q^2) \\ &\quad +(p-p')_\alpha(p-p')_\mu \tilde{G}_2^{(V)}(q^2)]+\cdots ,\end{aligned}$$

where the ellipsis represents other invariant structures which are automatically conserved and play no role in the determination of the spin content of these mesons. The quark spin content of these mesons $\Delta\Sigma^V$ is given by an expression similar to that of Eq. (5):

$$\Delta\Sigma^V = G_1^{(V)}(0)-\tilde{G}_1^{(V)}(0) = q^2\tilde{G}_2^{(V)}(q^2)|_{q^2=0}/2M_V . \tag{8}$$

If, as in the analysis of the proton's spin, we assume that $\tilde{G}_2^V(q^2)$ is dominated by the η' pole we find

$$\Delta\Sigma^V = \frac{\sqrt{2N_f}f_{\eta'}}{2M_V}g_{\eta' VV} . \tag{9}$$

The η'-V coupling constant is normalized by

$$\langle p',\lambda'|j_{\eta'}|p,\lambda\rangle = \frac{g_{\eta' VV}}{2M_V}\epsilon^{\mu\nu\rho\sigma}\epsilon_\rho^*(\lambda')\epsilon_\sigma(\lambda)(p+')_\mu(p-p')_\nu ; \tag{10}$$

$j_{\eta'}$ is the current coupling to the η' field. Again, corrections due to isospin and SU(3) breaking are small; these corrections are even smaller than in the nucleon case as, due to G-parity invariance, the isospin-violation corrections do not modify Eq. (9).

In order to proceed further we have to determine $g_{\eta' VV}$. Vector dominance relates the $\eta' VV$ vertices to the radiative decays $\eta'\to V+\gamma$ and $\eta'\to\gamma+\gamma$. The latter is especially appealing as the QED anomaly gives us a closed expression for this amplitude which agrees with experiment [8]. As we are neglecting the light-quark masses, we shall also neglect the 20° η-η' mixing [8]. From the QED anomaly and application of vector dominance to Eq. (10) the $\eta'\gamma\gamma$ vertices are

$$\begin{aligned}\mathcal{A}_{\eta'\gamma\gamma}^{(\text{QED anomaly})} &= \frac{\alpha}{2\pi\sqrt{2N_f}f_{\eta'}}\epsilon^{\mu\nu\rho\sigma}\epsilon_\rho^*(\lambda')\epsilon_\sigma(\lambda)(p+p')_\mu(p-p')_\nu , \\ \mathcal{A}_{\eta'\gamma\gamma}^{(\text{vector dominance})} &= \sum_V\left[\frac{e}{2\gamma_V}\right]^2\frac{g_{\eta' VV}}{2M_V}\epsilon^{\mu\nu\rho\sigma}\epsilon_\rho^*(\lambda')\epsilon_\sigma(\lambda)(p+p')_\mu(p-p')_\nu .\end{aligned} \tag{11}$$

In the above $eM_V^2/(2\gamma_V)$ is the amplitude for V-γ coupling. We compare the two expressions in Eq. (11), use nonet symmetry to equate the three η'–vector-meson couplings, and take the γ–vector-meson couplings in the ratio $\gamma_\rho^2:\gamma_\omega^2:\gamma_\phi^2=1:9:\frac{9}{2}$. Equation (9) and the above imply

$$\Delta\Sigma^{(\rho)} = \frac{3}{2\pi}\left[\frac{\gamma_\rho^2}{4\pi}\right] . \tag{12}$$

With [9] $4\pi/\gamma_\rho^2\approx 2$ we find $\Delta\Sigma^{(\rho)}\approx 0.24$

As mentioned in the previous paragraph, we can also use the decays of the η' into a vector meson and a single photon to determine the relevant coupling constants. This will avoid the need for nonet symmetry. Using vector dominance and Eq. (10) we obtain the amplitudes for such decays:

$$\begin{aligned}\mathcal{A}_{\eta' V\gamma} &= \left[\frac{e}{2\gamma_V}\right]\frac{g_{\eta' VV}}{2M_V} \\ &\times\epsilon^{\mu\nu\rho\sigma}\epsilon_\rho^*(\lambda')\epsilon_\sigma(\lambda)(p+p')_\mu(p-p')_\nu .\end{aligned} \tag{13}$$

Comparing with the experimental results for [10] $\Gamma(\eta'\to\rho,\gamma)$, $\Gamma(\eta'\to\omega,\gamma)$ and the limit on $\Gamma(\phi\to\eta',\gamma)$ we find $g_{\eta'\rho\rho}=1.27\pm0.04$, $g_{\eta'\omega\omega}=1.55\pm0.07$, and $g_{\eta'\phi\phi}<2.15$. The results are

$$\begin{aligned}&\Delta\Sigma^{(\rho)}\approx 0.25 , \\ &\Delta\Sigma^{(\omega)}\approx 0.30 , \\ &\Delta\Sigma^{(\phi)}<0.33 .\end{aligned} \tag{14}$$

How should we view this result? (i) We could accept the conclusion that only a small portion of the spin of the vector mesons is due to quarks. This is somewhat unnatural as at least a major part of the nucleon's spin is due to quarks [4]. (ii) Vector dominance could be inapplicable to this problem and the η'–vector-meson coupling is a factor of 3 larger than the estimates presented here. This is unlikely as all the radiative decays of the η' are consistent with vector dominance in that they give essentially the same coupling. (iii) The assumption that η' dominates the various form factors could be wrong

[3,11]. There is then the question of why the results on the nucleon are so close to the expectation that the spin of that particle is due to quarks. We cannot exclude the possibility that the dispersion relation for $q^2\widetilde{G}_2^{(V)}$ requires a subtraction, while the one for $q^2\widetilde{G}_2^{(P)}$ does not, or that the subtraction in the nucleon case is much smaller than in the vector-meson one. In terms of direct ghost couplings, we would have the situation where the ghost coupling to vector mesons is weaker than its coupling to nucleons.

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